Exercise 4

For each of the following integral equations, classify as Fredholm, Volterra, or Volterra-Fredholm integral equation and find its kind. Classify the equation as singular or not.

$$x + 1 - \frac{\pi}{2} = \int_0^{\frac{\pi}{2}} (x - t)u(t)\,dt$$

Solution

This is a Fredholm integral equation because both of the limits of integration are constant. It is of the first kind because the unknown function u appears only inside the integral. It's inhomogeneous because of the $x + 1 - \pi/2$ on the left side. It's not singular since neither of the limits of integration are infinite and the integrand does not become infinite in the interval of integration.